Centrality Algorithms

Rajesh Kumar Gupta

06/29/2026

1 Introduction

Centrality algorithms measure the importance or influence of nodes in a graph. Different centrality measures capture different notions of importance depending on the network structure and the application.

2 Common Centrality Algorithms

Algorithm	Measures	Typical Use Case
Degree Centrality	Number of direct connections	Finding highly connected nodes
Betweenness Centrality	Occurrence on shortest paths	Identifying bridges and bottlenecks
Closeness Centrality	Distance to all other nodes	Fast information dissemination
Eigenvector Centrality	Importance of influential neighbors	Measuring prestige and influence
PageRank	Random walk importance	Web search and citation ranking
Katz Centrality	Direct and indirect influence	Networks with indirect influence
Harmonic Centrality	Distance in disconnected graphs	Sparse or disconnected networks

Table 1: Common Centrality Algorithms

3 Degree Centrality

Degree Centrality is the simplest measure of node importance. It counts the number of edges connected to a node.

The normalized Degree Centrality is

$$C_D(v) = \frac{\deg(v)}{n-1}$$

where

- $C_D(v)$ is the Degree Centrality of node v.
- $\deg(v)$ is the degree of node v.
- n is the total number of nodes in the graph.

Nodes with larger degree values generally have greater immediate influence because they maintain more direct connections.

4 Betweenness Centrality

Betweenness Centrality quantifies how often a node lies on the shortest paths between other pairs of nodes.

It is defined as

$$C_B(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where

- σ_{st} is the total number of shortest paths from node s to node t.
- $\sigma_{st}(v)$ is the number of those shortest paths passing through node v.

Nodes with high Betweenness Centrality often serve as bridges connecting different communities.

5 Closeness Centrality

Closeness Centrality measures how close a node is to every other node in the network.

It is defined as

$$C_C(v) = \frac{n-1}{\sum_u d(v,u)}$$

where

- $d(v,u)$ is the shortest path distance between nodes v and u.
- n is the total number of nodes.

Nodes with higher closeness can spread information more efficiently throughout the network.

6 Eigenvector Centrality

Unlike Degree Centrality, Eigenvector Centrality considers not only the number of neighbors but also their importance.

It is based on the eigenvector equation

$$Ax = \lambda x$$

or equivalently

$$x = \frac{1}{\lambda} Ax$$

where

- A is the adjacency matrix.
- x is the vector of centrality scores.
- λ is the largest eigenvalue of the adjacency matrix.

A node connected to highly influential nodes receives a larger Eigenvector Centrality score.

7 PageRank

PageRank is an extension of Eigenvector Centrality developed by Google for ranking webpages.

The PageRank score is computed using

$$PR(v) = \frac{1-d}{N} + d \sum_{u \in M(v)} \frac{PR(u)}{L(u)}$$

where

- $PR(v)$ is the PageRank score of node v.
- d is the damping factor (typically 0.85).
- N is the total number of nodes.
- $M(v)$ denotes the nodes linking to v.
- $L(u)$ denotes the number of outgoing links from node u.

PageRank assigns greater importance to nodes receiving links from other highly ranked nodes.

8 Katz Centrality

Katz Centrality measures influence by considering both direct and indirect connections.

It is defined as

$$x = \alpha A x + \beta \mathbf{1}$$

or equivalently

$$x = (I - \alpha A)^{-1} \beta \mathbf{1}$$

where

- A is the adjacency matrix.
- α is the attenuation factor.
- β is a constant bias term.
- $\mathbf{1}$ is the vector of ones.

Unlike Eigenvector Centrality, Katz Centrality guarantees every node receives a non-zero score.

9 Harmonic Centrality

Harmonic Centrality is designed for disconnected graphs where Closeness Centrality may become undefined.

It is computed as

$$C_H(v) = \sum_{u \neq v} \frac{1}{d(v,u)}$$

where

- $d(v, u)$ is the shortest path distance between nodes v and u.
- If no path exists, the corresponding contribution is treated as zero.

Harmonic Centrality remains meaningful even when the graph contains disconnected components.

10 Comparison of Centrality Algorithms

Algorithm	Captures	Typical Complexity
Degree	Direct connectivity	$O(m)$
Betweenness	Brokerage and bridges	$O(nm)$ (Brandes)
Closeness	Reachability	$O(nm)$
Eigenvector	Prestige	Iterative matrix multiplication
PageRank	Random walk importance	Power iteration
Katz	Direct and indirect influence	Matrix inversion / iteration
Harmonic	Reachability in disconnected graphs	$O(nm)$

Table 2: Comparison of Centrality Measures

where

- n denotes the number of nodes.
- m denotes the number of edges.

11 Choosing the Appropriate Centrality Measure

Objective	Recommended Algorithm
Find the most connected node	Degree Centrality
Identify bridges between communities	Betweenness Centrality
Find nodes with fast reachability	Closeness Centrality
Measure prestige or influence	Eigenvector Centrality
Rank webpages or citations	PageRank
Include indirect influence	Katz Centrality
Analyze disconnected graphs	Harmonic Centrality

Table 3: Selecting a Centrality Measure

12 Applications

Centrality algorithms are widely used in numerous domains.

- **Social Networks**: Identifying influencers and community leaders.
- **Transportation Networks**: Detecting critical airports, stations, and road intersections.
- **Cybersecurity**: Finding vulnerable devices and critical infrastructure.
- **Citation Networks**: Ranking influential scientific publications.

- **Recommendation Systems**: Identifying important users and products.
- **Fraud Detection**: Detecting intermediaries in suspicious transaction networks.
- **Knowledge Graphs**: Ranking entities according to structural importance.

13 Conclusion

Centrality algorithms provide different perspectives on node importance within a graph. Degree Centrality measures immediate connectivity, Betweenness Centrality identifies bridges, Closeness Centrality measures reachability, Eigenvector Centrality and PageRank capture influence through important neighbors, Katz Centrality accounts for indirect influence, and Harmonic Centrality extends distance-based measures to disconnected graphs. Selecting the appropriate algorithm depends on the characteristics of the network and the analytical objective.